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The Distribution of the Stars of the Cape Photographic Durchmusterung. By A. M. W. Downing, D.Sc., F.R.S.

The publication of the C.P.D. is of peculiar interest as enabling us, for the first time, to attack the problem of stellar distribution on a sufficiently extended scale by a method which is free from many of the disabilities attaching to the older method of eye observing, although liable to other disabilities which affect the photographic method.

Thus the photographic plate is far more efficient than the eye in registering the stars to the same order of magnitude in the richer portions of the sky as in more sparsely occupied regions. On the other hand, the determination of a homogeneous system of stellar magnitudes by photography is a difficult and delicate operation.

Notwithstanding, therefore, the fact that M. Stratonoff has already published an instructive note on the subject in *Astron. Nachrichten*, No. 3710, I have thought that it would be of interest to astronomers to exhibit the general results of an analysis of the C.P.D. which I have made pending the publication of a further discussion of the registered stellar magnitudes which is expected from the authors, more especially as my work is carried out on somewhat different lines from those adopted by M. Stratonoff. In what follows the magnitudes of the catalogue are adopted throughout.

I have omitted the partial zone −18°, so that the stars whose distribution is here discussed all lie between −19° dec. and the South Pole (Equinox 1875·0).

The first, and of course by far the most laborious, part of the work was to count the stars of different magnitudes occurring in each zone. The magnitude-groups adopted were (1) 0 to 6·2, (2) 6·3 to 6·7, (3) 6·8 to 7·2, (4) 7·3 to 7·7, (5) 7·8 to 8·2, (6) 8·3 to 8·7, (7) 8·8 to 9·2, (8) 9·3 to 9·7, (9) 9·8 to 10·2, and (10) 10·3 and fainter. The numbers in the last group were formed by simply taking the differences between the sums of the first nine groups and the total numbers for each zone. The group is relatively too limited in numbers of stars to be of interest to us in the present connection.

In order to avoid the inconvenience of using large numbers I have exhibited the results of the counts as "density," or number of stars in one square degree.

The first table gives the mean density corresponding to each magnitude-group and the ratio (ρ), corresponding to one stellar magnitude computed from the expression

$$n \log \rho = \log D_{m+n} - \log D_m$$

where D_{m+n} is the density from magnitude 0 to magnitude $m+n$, and D_m is the similar density to magnitude m.

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It may be noted that the total number of square degrees comprised in the survey is 13,911·15, and that the aggregate mean density is 32·66. The total number of stars included in the table is 454,381.

TABLE I.

Range of Mag.	Mean Density.	Ratio.
0 to 6·2	·067	
6·3 „ 6·7	·058	3·48
6·8 „ 7·2	·158	5·13
7·3 „ 7·7	·346	4·94
7·8 „ 8·2	·861	5·61
8·3 „ 8·7	1·744	4·71
8·8 „ 9·2	4·619	5·90
9·3 „ 9·7	8·453	4·31
9·8 „ 10·2	12·254	3·07
10·3 and fainter	4·102	

No further reference will be made to the last magnitude-group.

It will be remembered that the authors claim that the survey is complete to mag. 9·2 only. The series of ratios given above would seem to substantiate this view, as there is a decided decrease after the above-mentioned limit is passed.

The next step in the discussion was to transform the places of the stars comprised in the different magnitude-groups to their galactic coordinates in order to investigate their distribution with reference to the galaxy. For this purpose the portion of the surface of the celestial sphere between −19° dec. and the South Pole was divided into zones of galactic latitude as follows:—

I. Latitudes −90° to −70°
II. Latitudes −70 to −50
III. Latitudes −50 to −30
IV. Latitudes −30 to −10
V. Latitudes −10 to +10
VI. Latitudes +10 to +30
VII. Latitudes +30 to +43½

The assumed position of the South Galactic Pole was R.A. 0^h 48^m, dec. −27½°.

The mean density in each zone was then found for each magnitude-group from the corresponding numbers of stars in each zone of declination and each hour of R.A.

The results of the transformation are exhibited in the following table, where mag. 6·5 means the magnitude-group 6·3 to 6·7;

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and similarly for the subsequent numbers given in the magnitude column.

Column VIII. gives the mean densities for the different magnitude-groups, excluding the stars in the equatorial zone of the galaxy, which are used in forming Column V.

TABLE II.

Galactic Latitude.

Magnitude.	I. −90° to −70°.	II. −70° to −50°.	III. −50° to −30°.	IV. −30° to −10°.	V. −10° to +10.	VI. +10° to +30°.	VII. +30° to +43½°.	VIII. Excluding the Galaxy.
To 6·2	·043	·053	·050	·077	·101	·066	·030	·059
6·5	·053	·062	·056	·063	·068	·046	·024	·056
7·0	·135	·125	·140	·176	·180	·149	·145	·153
7·5	·244	·268	·292	·352	·487	·353	·284	·312
8·0	·619	·634	·661	·884	1·346	·887	·679	·743
8·5	·917	1·023	1·219	1·869	3·035	1·727	1·386	1·431
9·0	1·489	2·107	2·981	5·116	9·232	4·343	2·865	3·500
9·5	1·618	2·592	4·249	9·464	19·512	7·995	4·689	5·771
10·0	2·593	3·412	5·058	11·354	29·808	15·503	6·280	7·997
Sums	7·711	10·276	14·706	29·355	63·769	31·069	16·382	
Sums to Mag. 9·2	3·500	4·272	5·399	8·537	14·449	7·571	5·413	
B.D.*	3·14	3·21	3·71	6·07	8·17	5·32		

We recognise here the ring of lucid stars lying nearly in the plane of the galaxy to which Gould and others have drawn attention. It will be remarked that the stars in magnitude-groups 6·5 and 7·0 are much more uniformly distributed than are the brighter stars, and that we must come down to magnitude-group 8·0 before we get as great a relative increase in density between the polar and equatorial zones of the galaxy as we find in the group of lucid stars.

The C.P.D. is not sufficiently extended in galactic longitude in this region to enable us to determine from it with accuracy the deviation (if any) of the ring of bright stars from the plane of the galaxy.

In studying the structure of our universe it is important to note the nearly uniform distribution of the stars of medium brightness.

The falling off in relative density after the magnitude 9·2 is passed, to which attention has been already drawn, as shown in Table I., is apparent throughout the zones of galactic latitude; equally in the polar zone (I.), the equatorial zone (V.), and in the mean densities, excluding the equatorial zone (VIII.). This

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* Seeliger's discussion of the Bonn *Durchmusterung* (see Newcomb, *The Stars*, page 249).

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must not, however, be taken as evidence of any thinning out of the stars of the fainter magnitudes here considered, but rather of the fact that we have passed the limit of magnitude to which the survey is complete.

The "Sums" show the aggregate densities in the different zones of galactic latitude for the stars down to mag. 10·2.

The "Sums to mag. 9·2" show the same down to the limit of magnitude to which the survey is assumed to be complete. The latter series is fairly comparable with the Bonn *Durchmusterung*, the results of which, as deduced by Seeliger, are given in the table. We recognise the greater relative aggregation of stars in the neighbourhood of the galaxy which appears in the photographic, as compared with the visual, survey, to which Kapteyn has drawn attention. Whether this arises from the greater efficiency of the photographic plate in the more crowded regions, or from the greater actinic activity of stars near the galaxy, appears as yet to be an open question. The C.P.D., however, agrees with the B.D. in assigning an ellipsoidal form to our universe of stars to the magnitude here considered, although pointing to a greater ellipticity of the ellipsoid, by reason of the greater relative extension towards the plane of the galactic equator.

Table III. gives the ratios of increase for one magnitude (as defined above for the corresponding quantities in Table I.) for the stars contained in the equatorial zone of the galaxy (I.), and for the residuum of stars, excluding this zone (II.). The latter series is comparable with the results obtained by Newcomb from his discussion of the star densities in certain non-galactic regions of the Cordoba DM., after the application of systematic corrections to the registered magnitudes to reduce them to the Harvard photometric scale (*Astronomical Journal*, No. 507). These are added in column III.

TABLE III.

Magnitude.	I. (Galaxy.)	II. (Non-galactic.)	III. (Non-galactic.)
6·5	2·80	3·80	3·44
7·0	4·26	5·43	
7·5	5·74	4·68	2·39
8·0	6·81	5·20	2·40
8·5	5·72	4·33	3·48
9·0	7·67	5·16	
9·5	5·52	3·70	2·42
10·0	3·53	2·77	2·55

The relatively large values obtained here, as well as in Table I., for the C.P.D., as compared with Newcomb's results, are remarkable, pointing to a systematic difference in the scale of magnitudes respectively adopted.

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The systematic corrections to the photographic magnitudes of the C.P.D. to reduce them to the visual magnitudes of Gould's Zone Catalogue, which are given by Kapteyn (*Annals of the Cape Observatory*, vol. v. p. 32), exhibit a similar discrepancy between the scales of the photographic and visual systems to that which is here disclosed. It will be remembered that Seeliger, in his discussion of the B.D. results, referred to above, found the value of the ratio in question to lie between 3 and 4 for the brighter stars. Seeliger's researches show that the ratio differs in different parts of the sky, and becomes exceptionally large in the galaxy itself, agreeing in this respect with the results for the relative values of the ratios given in columns I. and II. of Table III.

The publication of the authors' further discussion of the system of photographic magnitudes of the C.P.D. will be awaited with interest.

Note on the Principle of the Arithmetic Mean.

By H. C. Plummer, M.A.

1. In this note it is proposed to make a brief comparative study of certain lines of argument which are connected with the establishment of the arithmetic mean of a series of direct observations of a single quantity as the most plausible value to adopt. It has been suggested (by Encke, for example) that this position has been assigned to the arithmetic mean by experience, and even that the principle has an axiomatic character. This does not seem by any means to be the case. The arithmetic mean has always been, and probably always will be, adopted because it is by far the easiest function of the observations to calculate.* Also, although demonstrations of the principle may be wanting in rigour, its position has not been assailed by a rival more formidable in this respect or more acceptable on general grounds. If it be granted that positive and negative errors are equally likely to the same amount, it is obvious—and the formal remark has been made by Ellis †—that the arithmetic mean coincides with the true value in the limit when the number of observations is infinite. For if a is the true value, $x_1, x_2, \ldots$ the n observed quantities, and $e_1, e_2, \ldots$ the errors, then

$$\Sigma(x-a) = \Sigma e,$$

so that

$$a = \frac{\Sigma x}{n} - \frac{\Sigma e}{n} = \frac{\Sigma x}{n}.$$

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* With the exception of the Median, advocated by Edgeworth (*Camb. Phil. Trans.*, 1889, and *Phil. Mag.*, 1887), and by Estienne in his *Etude sur les erreurs d'observation*, Paris, 1890.

† *Camb. Phil. Trans.* vol. viii. p. 205 (1844).